Mathematics
QA
1
.M426

THE MATHEMATICS TEACHER

A MAGAZINE DEVOTED TO THE INTERESTS OF TEACHERS OF MATHEMATICS

Volume IV September, 1911 Number 1

CONTENTS

The Curriculum—Present Tendencies, Future Possibilities. J. T. Rorer . 1
What Results are We Getting from Graphic Algebra? Arthur Whipple Smith 13
Some Topics of School Mathematics of Special Importance to Those Students Who Expect to Study Analytic Geometry and Calculus. Walter B. Carver 21
Some Suggestions on Decreasing the Mortality in Our Geometry Classes. William R. Lasher 26
Some Experiments Testing the Most Rapid Method of Factoring the Type $ax+bx+c$. Fiske Allen 32
Notes on the Teaching of Mathematics in an Oberrealschule. M. O. Tripp 35
New Books 41
Notes and News 44

Published Quarterly by the

ASSOCIATION OF TEACHERS OF MATHEMATICS FOR THE MIDDLE STATES AND MARYLAND

LANCASTER, PA., SYRACUSE, N. Y.

NEW YORK, N. Y., PHILADELPHIA, PA.

Entered as second-class matter October 7, 1909, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879.

THE MATHEMATICS TEACHER

A MAGAZINE DEVOTED TO THE
INTERESTS OF TEACHERS OF MATHEMATICS

EDITOR

W. H. METZLER, Ph.D., F.R.S.E.
Syracuse University, Syracuse, N. Y.

ASSOCIATE EDITORS

EUGENE R. SMITH, M.A.
Polytechnic Preparatory School, Brooklyn, N. Y.
MAURICE J. BABB, Ph.D.
University of Pennsylvania, Philadelphia, Pa.

THE MATHEMATICS TEACHER is published quarterly—September, December, March and June—under the auspices of the Association of Teachers of Mathematics for the Middle States and Maryland.

The annual subscription price is $1.00; single copies, 35 cents.

Remittances should be sent to the Syracuse office by draft on New York, Express Order or Money Order, payable to The Mathematics Teacher.

THE MATHEMATICS TEACHER,
41 NORTH QUEEN STREET, LANCASTER, PA.
OR SYRACUSE, N. Y.

Just Published

The Teaching of Geometry

By DAVID EUGENE SMITH

Professor of Mathematics in Teachers College, Columbia University

$1.25

The appearance of this latest work upon the teaching of mathematics by Professor Smith is most timely and cannot fail to have a powerful influence not merely upon the work in geometry, but upon secondary education in general. The mathematical curriculum has been so severely attacked of late that a clear and scholarly discussion of the merits of geometry, of the means for making the subject more vital and more attractive, of the limitations placed upon it by American conditions, and of the status of the subject in relation to other sciences, will be welcomed by all serious teachers. This work meets the attack upon geometry with no hesitation, justifying for the subject a place in every scheme of education worthy the name, showing the fallacy of the ephemeral attempt to make it merely utilitarian in a narrow sense, admitting frankly that the old geometry contained certain matter that is not suited to present conditions, showing the various lines of application that may safely be used, and presenting the subject in the attractive light that has always characterized the work done by Professor Smith at Teachers College, Columbia University.

The Wentworth-Smith Geometry

The Wentworth Plane and Solid Geometry revised by George Wentworth and David Eugene Smith

In one volume: - - - - $1.30

In two volumes:

Plane Geometry - - $ .80

Solid Geometry - - $ .75

A revision that makes the leading geometry of twenty years even better than before.

It has been the aim to retain the remarkable simplicity and logical treatment of the Wentworth Geometry while improving it in content and appearance. The result shows The Wentworth-Smith Geometry to be the most usable and attractive geometry in this country. Among other changes are noticeable:

A more concrete introduction, acquainting the pupil at once with practical and interesting exercises.

A decrease in the number of propositions so that only the great basal theorems and problems are included.

A great increase in the number of simple problems.

An improvement in the arrangement of material.

With new cuts, new text, and new binding The Wentworth-Smith Geometry presents a book that answers every demand for an ideal modern course.

GINN AND COMPANY

BOSTON NEW YORK CHICAGO
LONDON ATLANTA DALLAS
COLUMBUS SAN FRANCISCO

WHEELER'S ALGEBRA

First Course in Algebra

By ALBERT HARRY WHEELER, Teacher of Mathematics in the English High School, Worcester, Mass.

Half leather, $1.15.

This modern book includes:

Three thousand mental exercises

The largest number of graded written examples

The best development of algebra from arithmetic

A lucid and early treatment of graphs

A through system of numerical checks

Proofs and reasons marked for omission as class work at the discretion of the teacher

Problems in Physics

THE SAME. BRIEF EDITION. (As far as Quadratics.)

Cloth, 95 cents.

Algebra for Grammar Schools

By A. H. WHEELER. Cloth, 50 cents.

LITTLE, BROWN & CO.

34 Beacon St., Boston **378 Wabash Ave., Chicago**

Practical Applications

of TRIGONOMETRY to Surveying, Astronomy, Navigation, Engineering, etc., are included in

Durell's Plane Trigonometry

Published 1910, Combined With

Durell's Spherical Trigonometry

Published 1911

CHARLES E. MERRILL CO.,

44-60 East 23rd St., New York City

Publishers of School and College Text-Books

Brooks's New Standard Algebra

By Edward Brooks, A.M., Ph.D.

A new book absolutely up to all modern requirements has been already adopted and is giving satisfaction in many of the best schools.

Brooks's Plane Geometry

Brooks's Plane and Spherical Trigonometry

Are the equal of any text-books of similar grade.

Furst's Mensuration

with special application of the prismoidal formula is the most compact and teachable book on the market.

Christopher Sower Company

Publishers

124 N. 18th St. Philadelphia

Scribner Mathematical Text Books

For the Grades

MOREY'S ARITHMETICS

Elementary Arithmetic . . . . . 50 cents
Advanced Arithmetic . . . . . 65 cents

For Grammar Schools, and Review Work in High Schools

Stevens & Butler's Practical Arithmetic . . . . . 65 cents

For High Schools and Colleges

MARSH & ASHTON'S MATHEMATICAL SERIES

Marsh's Elementary Algebra $1.00
Ashton's Plane and Solid Analytic Geometry $1.25
Marsh & Ashton's Plane and Spherical Trigonometry . $1.20
Marsh & Ashton's College Algebra $1.40

CHARLES SCRIBNER'S SONS

153-7 Fifth Avenue NEW YORK CITY

"CHAUTAUQUA"

Means These Three Things. Which Interests You?

A System of Home Reading

Definite results from the use of spare minutes. Classical year now in Progress. *Ask for C. L. S. C. Quarterly.*

A Vacation School

Competent instruction. Thirteen Departments; Over 2500 enrollments yearly. The best environment for study. Notable lectures. Expense moderate. July and August. *Ask for Summer School Catalog.*

A Metropolis in the Woods

All conveniences of living, the pure charm of nature, and advantages for culture that are famed throughout the world. Organized sports, both aquatic and on land. Professional men's clubs. Women's conferences. Great lectures and recitals. July and August. *Ask for Preliminary Quarterly.*

Chautauqua Institution, Chautauqua, New York

JUST PUBLISHED

Elements of Plane Trigonometry

By DANIEL A. MURRAY, Ph.D., *Professor of Applied Mathematics in McGill University.* 80 cents

This text-book is shorter than Dr. Murray's "Plane Trigonometry." The various historical and other notes have been omitted and the treatment of several topics condensed. Like Dr. Murray's other text-books it will be found thorough in fundamentals, accurate in statement and teachable in plan.

Monographs on Modern Mathematics

RELEVANT TO THE ELEMENTARY FIELD

Edited by J. W. A. YOUNG, Ph.D., *Associate Professor of the Pedagogy of Mathematics in the University of Chicago.* $3.00

A serious effort to bring within the reach of secondary teachers, college students and others at a like stage of mathematical advancement, a scientific treatment of some of the regions of advanced mathematics that have points in contact with the elementary field.

LONGMANS, GREEN, & CO., Publishers
4th Avenue and 30th Street NEW YORK

TEXT-BOOKS OF TRIGONOMETRY TABLES OF LOGARITHMS

By EDWIN S. CRAWLEY

Professor of Mathematics in the University of Pennsylvania

ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY. New and revised edition, vi and 186 pages, 8vo. Price, $1.10.

THE SAME, WITH FIVE-PLACE TABLES (as below), half leather. Price, $1.50.

This book is intended primarily for college use, but is used in many secondary schools also.

SHORT COURSE IN PLANE AND SPHERICAL TRIGONOMETRY. 121 pages, 8vo. Price, $0.90.

THE SAME, WITH FOUR-PLACE TABLES. Price, $1 00.

THE SAME, WITH FIVE-PLACE TABLES. Price, $1.25.

This book is intended primarily for use in secondary schools, but many colleges, in which the time allotted to trigonometry is restricted, have adopted it.

TABLES OF LOGARITHMS, to five places of demicals. Seven tables, with explanations. xxxii and 76 pages, 8vo. Price, $0.75.

These tables are much better arranged than most of the tables prepared for school use.

Orders, and requests for books for examination with a view to introduction, should be directed to

EDWIN S. CRAWLEY, University of Pennsylvania, Philadelphia

N. B.—In all cases specify by full title which book is desired.

IMPORTANT NEW BOOKS

Mechanics. By JOHN COX, M.A., F.R.S.C., Macdonald Professor of Physics in McGill University, Montreal, formerly Fellow of Trinity College, Cambridge. With four plates and numerous figures. 8vo. $2.75.

The Calculus for Beginners. By J. W. MERCER, M.A., Head of the Mathematical Department, Royal Naval College, Dartmouth. $2.00.

A Class Book of Trigonometry. By CHARLES DAVISON, Sc.D., Mathematical Master at King Edward's High School, Birmingham. $1.00.

A Course of Pure Mathematics. By G. H. HARDY, M.A., Fellow and Lecturer of Trinity College, Cambridge. $4.00.

Attention is also called to the following Series

The Cambridge Manuals of Science and Literature. Editors—P. GILES, Litt. D., and A. C. SEWARD, M.A., F.R.S. Already issued—**The Coming of Evolution,** by Prof. J. W. JUDD. **The English Puritans,** by Rev. JOHN BROWN. **Heredity,** by L. DONCASTER. **The Idea of God in Early Religions,** by Dr. F. B. JEVONS. **Plant-Animals,** by Prof. F. W. KEEBLE. **Cash and Credit,** by D. A. BARKER. Per vol. 40c. net.

The Cambridge English Classics. 15 vols. $1.50 each.

"Praise of appearance, type and paper cannot be too high."—*Athenæum.*

The Cambridge Historical Series. 16 vols. Edited by G. W. PROTHERO, Litt.D., LL.D.

Any of the above books sent for examination upon request.

Write for catalogues.

THE CAMBRIDGE UNIVERSITY PRESS

G. P. PUTNAM'S SONS

LONDON
24 Bedford St. Strand

NEW YORK
27 W. 23rd St.

The Stamp of Approval

Some Recent Adoptions

of

Gorton's High School Course in Physics

Malden, Mass.	Columbus, O.
Portland, Me.	Richmond, Ind.
Exeter, N. H.	Albion, Mich.
Rutland, Vt.	Watertown, Wis.
Plainfield, N. J.	Las Animas, Col.
Mt. Vernon, N. Y.	Cheyenne, Wyo.

D. APPLETON & COMPANY

NEW YORK CHICAGO

AN ALL-KNOWING SPECIAL TEACHER

ONE VOLUME containing the *pith* and *essence* of an authoritative library covering

Agriculture
Abbreviations
Architecture
Art
Astronomy
Bacteriology
Botany
Chemistry
Christian Names
Economics
Electricity
Fiction
Foreign Phrases
Forestry
Geology
Golf
Heraldry
Law
Mathematics
Mechanics
Medicine
Meteorology
Military Terms
Mineralogy
Mining
Music
Mythology
Nautical and Naval Terms

You are asked daily all kinds of questions. Where do you seek the final answers?

WEBSTER'S NEW INTERNATIONAL

THE MERRIAM WEBSTER

A New Creation, contains just the accurate, encyclopedic, up-to-date information you need.

Editor in Chief, W. T. Harris, Ph. D., LL. D., former United States Commissioner of Education.

400,000 Words defined. More than ever before appeared between two covers.

The only dictionary with the **new divided page,** "A Stroke of Genius."

6000 Illustrations. 2700 Pages.

An Encyclopedia. Type matter equal to a 15 volume set.

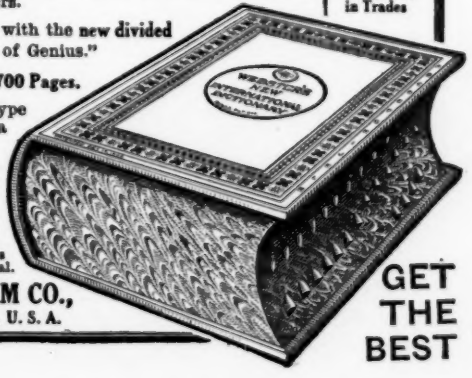
He Who Knows Wins Success.

WRITE for specimens of the new divided page, testimonials, illustrations, etc. FREE, a set of pocket maps if you mention this periodical.

G. & C. MERRIAM CO.,
SPRINGFIELD, MASS., U. S. A.

INCLUDES every field of the world's thought, action, and culture.

Philippine Island Words
Philosophy
Physics
Religious Terms
Scripture Names
Synonyms
Zoology
Biology
Geography
Words used in Trades



GET THE BEST

Advanced Texts in Mathematics

Church and Bartlett's Elements of Descriptive Geometry $2.25

Part I. — Orthographic Projections . . . 1.75

Lucidity and conciseness—text and figures in one volume — new problems — new figures. The old standard Church modernized.

Conant's Plane and Spherical Trigonometry. With Tables $1.20

Without Tables, $0.85. Tables Separate, $0.50.
Plane Trigonometry with Tables, $0.90.
Without Tables, $0.60.

Logical rigor—scientific accuracy—clear proofs. Unusual variety of concrete problems. Treats general subjects of mathematical investigation by means of trigonometric functions—is not limited to the solution of triangles.

Smith's Elementary Calculus. $1.25

Brief — definite — practical. For the technical student in other than engineering courses. Also for the general student who after trigonometry desires to divide a year between analytic geometry and elementary calculus.

Tanner and Allen's Brief Course in Analytic Geometry $1.50

Briefer than the authors' elementary course, but with the same rigor of proofs, careful analysis and the distinctive features that have made the earlier, larger work the standard in its field.

Merrill's Elementary Theoretical Mechanics. $1.50

Without calculus. Only knowledge of elementary algebra, plane geometry and plane trigonometry required. Limited to the most important principles and methods. For the two lower classes in college.

Catalogue of Books on Mathematics sent on request

American Book Company

New York **Cincinnati** **Chicago**

Compl. sets
Universal Lib. Service
7-7-38
36843

THE MATHEMATICS TEACHER

EDITED BY
W. H. METZLER

ASSOCIATED WITH
EUGENE R. SMITH MAURICE J. BABB

VOLUME IV SEPTEMBER, 1911 NUMBER 1

THE CURRICULUM—PRESENT TENDENCIES, FUTURE POSSIBILITIES.

BY J. T. RORER.

Introduction.

There is probably no better evidence of the increased interest in popular education than the present tendency to carefully scrutinize all parts of our educational machinery to note the defects and their remedy, to suggest improvements, and how to introduce them. The attitude of mind that conforms to the injunction, "Prove all things," is most healthful and should bring about better results. As teachers of mathematics we have nothing to fear from this critical spirit. We have proved all things from ancient times and our triumphal Q.E.D. has appeared over and over again. Shall we then not expect our own attitude of demanding proof to be a natural and reasonable one for others?

Mathematics has formed a part in all systems of education in all times, in varying amounts to be sure. In the days of our grandfathers the common school education was reading, writing and arithmetic. Modern development has brought about the introduction of many other studies. So that the modern course of study is exceedingly varied and complex, and we find our educational system from the bottom to the top criticized as being superficial and giving a smattering of too many subjects with a scholarly command of none. I believe we can all agree

upon this statement at least,—that the high school pupil of to-day faces too many studies at one time and that this dissipates his energy.

There has come into being within the last few years a new censor of education, The Carnegie Foundation for the Advancement of Teaching. The scholarship, the sincerity, the independent fearlessness of the management of this foundation, makes its utterances worthy of attention of all teachers. In the fifth annual report, the quality of high school and college training is considered from three points of view: (1) The testimony of the college entrance tests; (2) the testimony of employers of high school and college graduates; (3) the testimony of Oxford tutors. I will briefly give President Pritchett's conclusions from these three kinds of testimony.

Concerning college entrance tests, President Pritchett says: "A majority of the students entering college by examinations failed to fulfil the tests set up by these examinations. . . . In 1910, the College Entrance Examination Board examined about two thousand students; of this number about one half were not able to be rated as high as sixty per cent."

President Pritchett comments upon the testimony of business men: "While, therefore, it is impossible to summarize in a brief statement all that is contained in the criticism of business men concerning the performance of high school and college graduates, it may justly be said that business men would much prefer to receive, whether from high school or from college, the young man who has learned to do something well, and who has acquired the mental discipline which has come from this experience and who is ready to receive directions and follow them, than to take into their business either from high school or college, the ordinary representative, who comes with a little knowledge of many things, with no taste for the grind of business or of scholarship, and who lacks that grasp of his own mind which can come only from having acquired a mastery of some one thing."

In summarizing the testimony of the Oxford tutors, President Pritchett says, in part: "These observations . . . show also most clearly that in the majority of cases the (Rhodes scholarship) student finds difficulty in doing his work, arising out of

the superficiality and the diffuseness of his previous training in the American college, and the failure of this training to give him intellectual power. It is the characteristic weakness of the whole American life. The American has not yet got out from the spell of the pioneer days. He is alert, resourceful, but superficial. The pioneer stage itself has passed. Alertness and resourcefulness can no longer take the place of thorough training and careful preparation. To-day the great opportunity of the secondary school and of the college is to cure this weakness, not to minister to it."

If necessary, much corroborative testimony might be presented to establish the fact that our present curricula, as applied to the individual student, are absurdly diffuse and that this diffuseness has had for many years a weakening influence on school and college work. Teachers everywhere have noted this unfortunate condition; it is only recently, however, that a revolt has begun against this wrong.

College Entrance.

College entrance requirements have covered too many subjects, and the school courses have included even a wider range. In recent years the secondary schools have responded to the popular demand for courses that applied closely to the present and the future needs of the pupils along vocational lines, manual training, bookkeeping, stenography, and typewriting, agriculture, sewing, cooking, etc. We can give these manual subjects credit for breaking the fetters of tradition and opening the way of reform. Clearly, if a considerable part of the five or six hours of the high school day is given for such vocational work, the remaining time is insufficient for the ground prescribed by the colleges. In the east, practically no credit has been given for such vocational work, hence pupils taking such courses have been unfairly handicapped when asking for college admission.

During the past year, several official reports have been made against these and other unhappy conditions which I have not taken time to mention. Among these reports let me note the following: Resolutions of the Secondary Department of the National Educational Association; Resolution of the Depart-

ment of Manual Training and Art Education, N. E. A.; Resolutions of Department of Business Education, N. E. A.; Report of the Committee of the New York High School Teachers' Association; Report of the Committee on College Entrance Requirements of the Federation of Teachers of the Mathematical and the Natural Sciences; Report of the Boston Head Masters' Association.

The demands of these reports and resolutions are chiefly the reduction of the number of required subjects, and the recognition of a wider range of subjects, especially of the vocational subjects.

Some response has already been made. Harvard is starting an influence in the right direction in her new entrance requirements which specify examinations in four subjects as follows:

(A) English.

(B) Latin, or, for candidates for the degree of S.B., French or German.

(C) Mathematics, or physics, or chemistry.

(D) Any subject (not already selected under (B) or (C)) from the following list:

Greek,	History,	Physics,
French,	Mathematics,	Chemistry.
German,		

In addition to facing these four examinations the candidate must produce a statement showing: "(*a*) A four years' course of study; (*b*) that his course has been concerned chiefly with languages, sciences, mathematics, and history, no one of which has been omitted; (*c*) that two of the studies of high school program have been pursued beyond their elementary stages, *i. e.*, to the stage required by the present advanced examinations of Harvard College or the equivalent examination of the College Entrance Examination Board."

These requirements make it possible that no more than five subjects occupy the attention of the students at one time. This does not mean that fewer subjects must be taught to the individual and necessarily implies that each boy and girl shall select under guidance those subjects which seem to be best adapted to his present and future needs.

Pedagogic Values.

The agitation for a change in the curriculum leads to an examination of subjects and methods of teaching which shall reveal their pedagogical values.

In recent years our teachers of mathematics have divided into two camps: first, those who claim that mathematics is only justified in the curriculum for its practical application; second those who claim that its cultural value, its power to train the reasoning faculties, is its greatest argument for existence in the curriculum.

If one's lot is to teach a group of specialized students, a group of engineers, for instance, he may sound the practical application idea with considerable force. If, however, it is your lot to believe in algebra, geometry, trigonometry, etc., *for the multitude,* how can you claim that your average student will ever in his mature years square a binomial or a hypotenuse, or obey the law of sines? Accordingly, this latter group of teachers who are aware of actual conditions and who are honestly presenting mathematics without extravagant claims for its daily application, must examine carefully the faith that is in them. Is power cultivated in mathematical studies transferable to the non-mathematical problems of life? In what way will algebra and geometry make a better salesperson, physician, nurse or journalist?

The present trend of pedagogy is to discover tests of efficiency in producing thought power. Our opponents claim that mathematical training at best but develops certain brain cells which, when they respond, produce the successful mathematical thinker; that these cells are narrow specialists and their training is as useless for other kinds of thinking as is the use of chest weights to produce technique on the piano. Dr. George E. Dawson, in *Popular Science Monthly* (September, 1910) says: "The intellectual culture derived through standarized branches of education, as mathematics and Latin, for example, instead of having a general mental economy for the innumerable young men and women who study them, in reality becomes parasitic in the nervous and mental life, and this is a cause of wasted energy and possibly, of disease."

Dr. Dawson then refers to the over-trained athlete typical of

our teams whose subsequent life is actually shortened by a strained heart or weakened lungs and claims that "his vitality is reduced through parasitic muscular culture."

Such an argument takes into account but few cases of misdirected and narrow specialization. No intelligent physical director will to-day endorse such training. It is common observation that where *one* suffers from over training hundreds, perhaps *thousands* suffer from insufficient training. To compare ordinary school and college courses in languages and mathematics to such physical over-exertion and straining is manifestly absurd. We must also dissent from the opinion that mathematical culture and power are not transferable to other phases of mental action. The weight of opinion, past and present, is strongly on our side. Mathematicians are sometimes cartooned as freaks—out of place in the world's work. Such is not the teaching of history; nor, as a class, are the members of the American Mathematical Society, for instance, in any way more abnormal than the members of any other group of educated people.

The Tests of Modern Psychology.

It may seem presumptuous for one untrained in modern psychology to criticize the opinions and evidence of the masters of that science. Nevertheless, I wish to make a few comments on some of the statements of the quantitative psychologists, particularly on the topic of correlation.

Correlation is defined "as a tendency of two or more traits or capacities to vary together."* It is what we might term *functionality* of ability, capacity, or trait.

"A correlation is indifferent or *zero* if the existence or variation of one trait is totally unrelated to that of the other. In *perfect positive* correlation (direct linear variation) the correlation is unity or 1.00. . . . In actual psychological investigation, at least when functional correspondences are under investigation, we have commonly to deal with some intermediate degree of correspondence, and r, the correlation coefficient, assumes therefore the form of a decimal lying between 0 and

* Whipple, "Mental and Physical Tests," p. 28.

+ 1.00 for positive, and 0 and — 1.00 for negative correspondence."*

The formulas used in the computation of r by the product-moments method of Pearson are:

$$r = \frac{\Sigma xy}{n\sigma_1\sigma_2}$$

in which x = the residual of a single observation of the first kind from the mean of all the observations of the first kind;

y = the similar residual from observations of the second kind;

n = the number of individuals observed;

$$\sigma_1 = \sqrt{\frac{\Sigma x^2}{n}}, \qquad \sigma_2 = \sqrt{\frac{\Sigma y^2}{n}},$$

where Σx^2 = the sum of the squares of the residuals of the first kind; and Σy^2 = the sum of the squares of the second kind.

In order to estimate fairly the results of correlation studies pertaining to mathematics, a few correlation results are given in other subjects.

Test of 50 boys (Whipple), strength of grip comparing right and left hands, $r = .93 \pm .02$.

Correlation of stature of father and son (Pearson), $r = .396$.

Stature of brother and brother (Pearson), $r = .391$.

The problem of the correlation between efficiency in mathematics and in other subjects is commanding the attention of several quantitative psychologists. Thus far, the results of these investigations are by no means in agreement. Rietz and Shade† have found a decided positive correlation between efficiency in mathematics and efficiency in the natural sciences and in languages.

Mathematics and foreign languages, $r = .476$.

Mathematics and natural science, $r = .440$.

Below, in column 1 are the results from study of about 500 pupils in sixteen high schools by W. P. Burris; in column 2 are the results of 232 New York Regents' Examinations, but not

* Whipple, "Mental and Physical Tests," pp. 28 and 29.

† *University of Illinois Bulletin*, Vol. VI., No. 10.

more than 108 cases in any one computation, by Brinckerhoff, Morris and Thorndike.*

Correlation Between	1.	2.
Mathematics and English	.39	.09
Mathematics and Latin	.40	.31
Mathematics and history	.33	.26
Mathematics and science	.41	.07
Algebra and geometry	.45	—

Thus, taking mathematics and science $r = .41$ (Burris) for an illustration, the meaning is if we take any group of mathematics students who rank x above or below the average, this group will on the average rank $.41x$ above or below the average in science. Suppose the class average in both subjects is 70 per cent., then the boy who obtained 100 in arithmetic might expect 82 per cent. in science.

The authors of these studies do not put any great stress on the exactness of their results. They seem to believe them to be but indications of correlations that may or may not exist. In this connection, Whipple says: "As a matter of fact, some of these published statements of correlation are actually wrong; correlations do not exist where they have been affirmed or do exist where they have been denied."

Burris says: "So far as the actual results are concerned (referring to his own results as given above), everything depends upon the reliability of the data with which the method deals; and . . . there are numerous reasons for questioning the value of the data so far used."

Observations on Psychological Testimony.

Now it seems to me that these conclusions can be drawn from the recent studies of the quantitative psychologists in regard to the correlation of mathematics and other subjects in the curriculum:

1. The psychologists are not in agreement among themselves on this question.

2. The conclusions are drawn from insufficient data. In physical science, in astronomy for instance, where fixed laws are postulated, and personality, except of the observer, does not

* Columbia University, Contributions, Vol. XI., No. 2.

enter the problem, often thousands of observations are used as a basis of a single result; in sciences, like psychology, where any result may be a function of so many phases of personality and environment, the basic data should be correspondingly increased. In the studies quoted above, the number of observations is far too small.

3. Some of the psychologists are expecting too high a value for the coefficient of correlation. Few practical teachers would say that because a boy did excellent work in algebra that he would do excellent work in Latin; but rather that in all probability he would do better work in Latin than a boy who failed in algebra.

Before leaving this topic which may seem to some an unwarranted digression from my main title, but which I regard as vitally connected with the whole curriculum and especially with mathematics, unless we become teachers of applied mathematics only, let me suggest the we enter the field of experimental psychology and personally investigate questions that bear upon our work.

It is tragic to think of investigators using the method of least squares, to which so many eminent mathematicians have contributed, to question the value of mathematical teaching for the general purposes of life.

Some Results of This Criticism

What will be the results of the present educational agitations and criticisms? Perhaps they may be summed up in this one word *correlation* better than any other word.

1. There must be a careful and intelligent correlation within the mathematical field. We now admit that there are no fixed boundaries between the divisions of mathematics. Text-books are appearing welding together arithmetic, algebra and geometry. This idea is only in its incipiency and needs careful development. We need to encourage the correlative temperament. Isolated facts are difficult to retain and cannot be applied until they are correlated. In examining candidates for teachers' positions in some of the higher schools of Philadelphia, I have been astonished at the evident neglect of summarization and correlation on the part of young teachers.

2. There must be a discriminating correlation with the fields of applied mathematics. Too few of our teachers have at command even one field of application, such as physics, statistics, actuarial science, architecture, or engineering. One of the chief benefits of the modern movement is that many teachers are now studying the applications of elementary principles.

3. There must be a broad correlation with the non-mathematical studies such as the languages and history.

All good mathematics will translate into good English. If Latin and Greek are occupying the attention of the pupils, the etymology of the words used reveals their meaning. I have always found a class of Greek students to be interested in the propositions of Euclid in the original Greek. The history of mathematics will find a still larger place in our class rooms. It greatly adds the element of human interest.

4. There is an esthetic and ethical value of mathematics that is yet to be developed and appreciated.

The teacher who is not awake to the beauties of the science can never do first class work. The simplicity and the completeness of a proof or of a construction, the wonderful properties of a curve, or of a series, should awaken the enthusiasm of both teacher and pupils. For this reason, the early introduction of some more advanced topics is advisable even with secondary classes. A determinant solution of simultaneous linear equations in two or three variables awakens a lively interest; the geometry of Feuerbach's circle gives beautiful exercises on Books I., II. and III.; the fundamental properties of the parabola, ellipse and the cycloid and their applications will enthuse many a high school boy to press on to the Better Land of Higher Mathematics.

Character building is properly considered to be the chief work of the modern teacher. Mathematics is not wanting in its ethical bearing. "God is continually geometrizing," said Plato, expressing his idea of the dignity and beauty of the reason. The care with which we distinguish right from wrong in method and result, the habit of self-scrutiny in testing work, the element of fairness and equity exemplified in the equation,—all these and many other phases of our subject furnish fruitful ethical suggestions. As a climax, perhaps, we might place the

reverence for truth which mathematics should develop. The final appeal is never to personal opinion, nor to the authority even of the wisest, but to the simple truth as judged by the reason.

"As pure truth is to the pole star of our science," says Simon,* "so it is the great advantage of our science over others that it awakens more easily the love of truth in our pupils."

Another German writer has well put our claim—"Who does not know mathematics, and the results of recent scientific investigation, dies without knowing *truth*."

Conclusions.

However crowded the curriculum may be, mathematics will continue to occupy an important part of the educational program. The verdict of centuries of pedagogical progress cannot be reversed by the loud acclaims of a few radical agitators.

The mathematical teaching of the future must put still more stress on power of observation, reasoning, and invention. Our material must be rearranged with a view to presenting the vital parts as soon as possible. We are so accustomed to our traditional order that few of us conceive that other orders of presentation are possible. How little preliminary mathematics, for instance, is requisite to the solution of right and oblique triangles! Should logarithmic computation be confined to trigonometry?

Our courses in the future must also give more attention to the instruments of our craft. Why should we neglect the slide-rule, the planimeter, the transit and other forms of measuring instruments?

There must be an increasing confidence and respect among teachers, and among groups of teachers. The examination system is one of the obstacles which to a large extent interfere with harmony and mutual helpfulness. A brand on a piece of hardware, a guarantee of a reputable commercial house, the opinion of a learned jurist, carry weight and command respect. Why should not the opinion and the certificate of a teacher or of a school have meaning and force? The examination will always have a place but its place has been and is greatly magni-

* "Mathematical Unterricht."

fied, the *testimony* of the trained teacher is far more important. The future will magnify the *opportunity* and the *dignity* of the teacher. There will be more liberty of action, hence more responsibility. The conditions under which we work are becoming more and more diverse, hence no single method or text can be universal or general. Each must face his problems with discrimination, and each earnest worker should have the respect of all.

WILLIAM PENN HIGH SCHOOL FOR GIRLS,
PHILADELPHIA, PA.

In this world of imperfections we gladly welcome even partial intimacies. And if we find but one to whom we can speak out our heart freely, with whom we can walk in love and simplicity without dissimulation, we have no ground of quarrel with the world or God.

When we have fallen through story after story of our vanity and aspirations, and sit ruefully among the ruins, then it is that we begin to measure the stature of our friends; how they stand between us and our contempt, believing in our best.—*Robert Louis Stevenson.*

WHAT RESULTS ARE WE GETTING FROM GRAPHIC ALGEBRA?

By Arthur Whipple Smith.

I feel that I should explain to you that my acquaintance with the work of the secondary schools is entirely second hand and it may be that my ideas on the subject of graphics in secondary work are colored by what I may wish were possible instead of being entirely true to the facts. In my own experience as an instructor of freshmen I have found but little evidence indicating previous instruction in graphics and frequently the subject is thrust suddenly upon a freshman by the immediate needs of his college work. In many cases it is looked upon as only another novelty introduced to make college mathematics a thing to be dreaded and avoided if possible. I assume that it is proper for an instructor in first-year college work to expect a greater or less degree of familiarity with graphics on the part of incoming students and from this standpoint consider the question as to what may be gained by the study of graphics in connection with all branches of elementary mathematics. The subject should appeal to teachers of mathematics for at least three reasons, first, it is the simplest of our many symbolisms for magnitudes and in the order of nature precedes all the others: second, it often appeals to certain pupils who would otherwise be uninterested: third it affords connecting links among all branches of mathematics.

As to the first reason, Dr. Taylor, of Colgate University, has recently been giving some attention to the subject in connection with his work in the teachers' course and with his permission I shall quote from him.

"We have in general two methods of expressing our concepts and thoughts, viz: the graphic and the symbolic. The traveler makes known his adventures graphically by pictures and drawings as well as symbolically by words. To give clear and vivid ideas of magnitude the statistician represents them graphically by lines as well as symbolically by numbers. In the

order of nature the graphic method precedes the symbolic. The young child knows the picture horse before he knows or uses the word "horse." He often learns and can repeat illustrated poems when without the illustrations to interest and guide him he could not repeat a single word. The human race by the graphic method cultivated and perfected elementary geometry nearly two thousand years before the study of geometry by the symbolic method was dreamed of. When graphic geometry had been perfected the symbolic sciences of arithmetic and algebra were in their infancy. These facts would suggest that in our mathematical instruction to the young at least we should use the graphic method before we do the symbolic, but the reverse is our practice, the graphic being taught last, if at all. Pupils in arithmetic often do not suspect that there is any other *mathematical* way of expressing magnitudes or size than by numbers and when they are taught to represent distances and other magnitudes by the length of lines, *i. e.*, graphically, they generally fail to understand that the method is mathematical. Before a pupil begins the study of algebra, even while he is pursuing arithmetic, the graphic method of representing magnitudes should be made familiar as a method of mathematics. At this early stage the mathematical laboratory is the drawing room equipped with graded weights, spring balances, and other needed apparatus, where the pupil should learn how to represent distances, forces, and other magnitudes by lines or other drawings. For example, after he has decided how long a line he will use to represent the force needed to lift a weight of one pound, he should express by lines the forces needed to lift two pounds, three pounds, etc. He should also learn map drawing, not simply by copying the work of others but by making maps from given data, or better still, from data which he has himself collected. This graphical method would interest many to whom the symbolism of numbers is distasteful and at the same time might prove to be an excellent exercise in mathematics if properly done."

We thus see that the subject may quite properly be begun in the earliest of mathematical work and in a few particular instances I shall endeavor to point out concrete applications in geometry and algebra.

In plane geometry the usual theorems on areas, *e. g.*, those concerning the rectangles of sums and differences of lines, may be taken as the graphic forms of the algebraic theorems on the squares and products of sums and differences. Some writers on geometry introduce these algebraic theorems as corollaries to the geometric theorems.

In algebra the subject of surds usually proves a stumbling block to those who see no concrete meaning to such expressions as the square root of two, especially after he has been told that these are neither whole numbers or fractions. The pupil is quite likely to believe that such numbers are only the result of mathematical gymnastics. But here we find the advantage of graphics. We may recall the theorem on the square of the hypothenuse of a right triangle and form a figure by first constructing a right triangle with the legs each equal to one. On this triangle we build a second having as one leg the hypothenuse of the first triangle and the second leg equal to one. This process we continue indefinitely. It appears then that the lengths of the successive hypothenuses are to be represented symbolically, by the square roots of the successive integers 1, 2, 3, 4, 5, etc. The triangles are concrete, so is the hypothenuse of each, and the need of the symbols representing their lengths must be at once conceded.

In beginning algebra we encounter the difficulty of presenting to the beginner the idea of positive and negative numbers. That this is often poorly done is recognizable by the frequent statement on the part of pupils that all numbers in arithmetic are positive, thus failing to see that positive number implies the existence of negative number. Whenever I meet this idea in the mind of a pupil I ask him to define the meaning of the word *cold* if he were to assume that the entire universe was and always had been at a uniform temperature. In other words positive and negative are but names which we give to numbers which are used to denote the sizes of quantities which in addition to *size* possess the property of *oppositeness, i e.*, of mutual destruction when combined. Now a simple application of graphics may be used to fix these new ideas with clearness. We assume that the pupil has already learned in arithmetic to represent the relative sizes of magnitudes by both number symbols

and line segments. By introducing the subject of forces from familiar everyday illustrations we obtain the addition of direction to the lines used, including the idea of opposite directions. The next inquiry is how can both the directions and sizes of forces, which may be opposing forces, be represented by number symbols as well as by directed lines. The answer is that the size may be represented by the old arithmetic or size numbers, but that the idea of oppositeness requires the invention of *new* number symbols. Thus the need of numbers to represent both the size and the oppositeness of such forces introduces by definition two kinds of number, one called positive and the other negative, each possessing, in common with the other, *size* or *arithmetic value*. The rules for the addition and subtraction of such quality numbers are easily illustrated or even surmised from examples involving the composition of forces acting in either the same or opposite directions. A further help in this direction and one which has important uses later is the scheme of geographic latitude and longitude, in which the notations N and S (or E and W), are replaced by the use of positive and negative numbers respectively.

It is impossible to give any concrete meaning to a single equation in two unknowns if we limit the treatment to the question of its solution but it is possible to do so by means of its graphic representation, in the discussion of which we find geometric meanings not only for the equations individually but as well for all the possible peculiarities which may arise in discussing them simultaneously. The simplest introduction to this graphical study is a review of geographical latitude and longitude. Noting here the oppositeness as expressed by N and S, E and W, the pupil thus learns that any point on the earth's surface (or on a plane, if we draw two perpendicular lines to replace the equator and principal meridian), can be represented by two *quality-numbers* denoting the distances and directions of the given point from the two fixed reference lines. The converse—that every two such numbers define the position of a point—easily follows.

If now we recall the locus idea of geometry, *i. e.*, that the locus is an aggregate of points bearing some definite relation to each other or to some given configuration, it is easily inferred

that the pairs of numbers which represent these points must bear some definite relation to each other. If now we lead the pupil to see that there is an unlimited number of solutions, *i. e.*, pairs of numbers, for each linear equation in two unknowns and recall to him that each of these pairs represents, or is pictured by, a point we have opened the way to the graphics of the equation. It is evident that these many pairs of numbers cannot be chosen at random for each such pair must satisfy the equation and hence the points which picture these pairs cannot be chosen at random but are subject to a condition, *i. e.*, they form a locus of some sort. There are then two ways of expressing a locus, either by an equation (*i. e.*, symbolically) or by a graph. If these conclusions be granted we have the following simple theorems:

Every equation in two unknowns represents a relation connecting pairs of numbers (its solutions). The graphs of such related pairs of numbers form a geometric locus.

By a solution of two equations in two unknowns we mean that pair (or pairs) of numbers which is common to both, and hence geometrically such a common solution must represent the point (or points) which are common to the two loci corresponding to the equations, *i. e.*, their intersections.

If two equations have no common solution, their loci do not intersect. If two equations have a finite number of common solutions, their loci have the same number of points of intersection. If two equations are equivalent, their loci coincide. A little thought will show the graphic meaning for every condition which may arise in the algebraic solution of the equations.

I do not think it will prove too difficult for those of the class who show any curiosity at all as to the character of these loci to understand that the linear equation in two unknowns represents a straight line. The following proof requires only the most elementary knowledge of the equation and a little geometry.

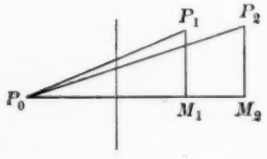
We start with the general equation $ax + by = c$ and solve it for y, obtaining

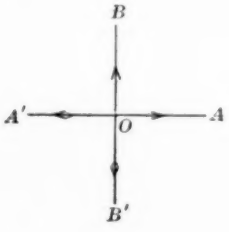
$$y = \frac{c - ax}{b}.$$

For simplicity we represent the fractions a/b and c/a by

the letters $-m$ and d. The equation now takes the form $y=m(x-d)$. One solution of this equation is (d, o) the picture of which we call P_0. We suppose that x_1y_1 and x_2y_2 are two other solutions the pictures of which we represent by P_1 and P_2.

We now join the points P_0 and P_1 and P_0 and P_2 by straight lines forming with the perpendiculars M_1P_1 and M_2P_2 two right triangles.





From the figure we find $P_0M_1=x_1-d$,
$P_0M_2=x_2-d$,
$M_1P_1=y_1$,
$M_2P_2=y_2$,

and by using the given equation we obtain

$$\frac{M_1P_1}{P_0M_1}=m=\frac{M_2P_2}{P_0M_2}.$$

Hence the two right triangles are similar and angles $M_1P_0P_1$ and $M_2P_0P_2$ are equal, *i. e.*, the straight line joining the two points of the locus P_0 and P_1 passes through any third point P_2. The locus is therefore a straight line.

Some may say that pupils in secondary work are not prepared for such reasoning. My own opinion is that such work is preferable to spherical trigonometry and the study of series which subjects may freshmen claim to have had.

And furthermore the future study of the equation will naturally bring to the minds of most pupils a desire to know more of the graphical side, *i. e.*, of analytical geometry, a result which is certainly much to be desired. If the teacher can point out to them as facts (the proof of which they can learn later)

that the heavenly bodies which move about the sun always travel in one of two kinds of orbits, viz: ellipses or parabolas, that these curves are called conic sections because they may be cut from the right circular cone, that they are to be studied later in connection with the study of equations of the second degree in two unknowns, I feel certain that the algebraic equation will at once become a thing of importance in the eyes of most pupils.

For those who teach the subject of imaginary and complex number there is no better way of approach than the graphic. We consider the problem of forces at right angles to each other. The use of positive and negative numbers is of course sufficient for representing the opposite forces in either of the lines of direction but the problem here calls for numbers of two kinds which cannot by ordinary processes of addition and subtraction be combined into a single number. The definition of such new numbers may be made as follows. Let equal forces at right angles to each other be represented by the four lines OA, OA', OB, OB'.

Let the magnitude of these equal forces be represented by the *arithmetic* number a. We then designate OA by the *algebraic* number ^{+}a and OA' by ^{-}a. According to the definition of positive and negative numbers these may be written as $^{+}1 \cdot a$ and $^{-}1 \cdot a$ respectively, a notation which suggests the possibility of a similar one for representing OB and $OB.'$ We then represent OB and OB' by the symbols $^{+}i \cdot a$ and $^{-}i \cdot a$ respectively. As an arbitrary notation this might be sufficient but we desire also to find some relation between these two kinds of number symbols.

The vector OA becomes OB graphically by a counterclockwise revolution of 90° and symbolically by using the factor i. Also OA becomes OA' by a similar revolution of 180° and symbolically by using the factor -1. If we attempt to identify this graphic process with the symbolic we are forced to set the 180° in correspondence to the factor -1 and 90° in correspondence to the factor i. But since 180° is formed by two consecutive 90° revolutions, the factor -1 should be identical with the successive applications of i and i, *i. e.*, of i^2. Thus we obtain $-1 \equiv i^2$ or $i \equiv i\sqrt{-1}$, a number scheme which will be found to be quite self-consistent and from which the theory of imaginaries is easily derived.

The few examples of graphics which I have given are, I think, sufficient to point out that this method cannot help but weld together subjects which in the minds of too many are quite distinct. I often wonder if many of those who so dislike the subject in general would not change their opinion if they were to keep at it until they reach analytics, for example. Algebra is so often taught as mere machinery for grinding out answers with grades accordingly, that I do not wonder at the dislike for the subject. We must seek to make mathematics interesting without losing any of its rigorousness and one of the simplest ways is to knit all its branches together early in the course and keep before the pupil, so far as possible, a hint as to what he is approaching. For the teacher I know no easier way than the graphical one.

COLGATE UNIVERSITY,
HAMILTON, N. J.

There is an idea abroad among people that they should make their neighbors good. One person I have to make good: myself. But my duty to my neighbor is much more nearly expressed by saying that I have to make him happy—if I may.

A man who has a few friends or one who has a dozen (if there be any one so wealthy on this earth), cannot forget on how precarious a basis his happiness reposes; and how by a stroke or two of fate—a death, a few light words, a piece of stamped paper, or a woman's bright eyes—he may be left in a month destitute of all.—*Robert Louis Stevenson.*

SOME TOPICS OF SCHOOL MATHEMATICS OF SPECIAL IMPORTANCE TO THOSE STUDENTS WHO EXPECT TO STUDY ANALYTIC GEOMETRY AND CALCULUS.*

By Walter B. Carver.

The writer has, for some time, been teaching analytic geometry and calculus to engineering freshmen; and it is his aim, in this paper, to call attention to certain subjects in which his students have seemed to be pretty generally deficient. The topics have been selected somewhat at random, and probably others matters of equal importance have been passed over.

In geometry the essential thing is the development of the student's ability to reason accurately and logically, and this presents one of the big problems of school mathematics—a problem too big for discussion in such a paper as this. In addition to this mental training, the student should acquire a thorough familiarity with the rules for areas, volumes, etc., and with the rules of proportionality of similar figures.

The first essential in algebra is a thorough drill in manipulation. The student should, by much practice, acquire *facility* in factoring, in the insertion (as well as the removal) of parentheses, and in handling simple and complex fractions and radicals. He should not be so easily discouraged as he is when, for example, he has occasion to determine whether or not the equation

$$(5-\sqrt{2})x+(4\sqrt{2}-3)y+1-7\sqrt{2}=0$$

is satisfied by the values

$$x=-\frac{5\sqrt{2}}{2} \quad \text{and} \quad y=\frac{12+3\sqrt{2}}{2}$$

In such work as this, students should learn to save time, without any loss of accuracy, by doing more of the details mentally

* Abstract of a paper read before the Syracuse Section.

and writing less. For example, to solve the equation

$$2x^2 - 3xy + 4y^2 - 2x + 7y - 11 = 0,$$

for x in terms of y, only three steps are necessary. Nothing need be written but

$$2x^2 - (3y + 2)x + (4y^2 + 7y - 11) = 0,$$

$$x = \frac{3y + 2 \pm \sqrt{9y^2 + 12y + 4 - 32y^2 - 56y + 88}}{4}$$

$$= \frac{3y + 2 \pm \sqrt{-23y^2 - 44y + 92}}{4}$$

This example suggests several other things. The student should understand certain algebraic language—he should know what is meant when he is asked to "solve an equation for x in terms of y" or to "arrange an equation as a quadratic in x." Here also is one of the cases where we need the process of inserting parentheses.

As a regular working method for quadratic equations, the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

should be used. The method of completing the square should be taught, of course, and the formula derived by means of it; but it is wearisome to use this method when one has occasion to solve many quadratic equations, as in analytic geometry. It is also important that the relation between the discriminant $b^2 - 4ac$ and the nature of the roots of the equation should be more thoroughly mastered.

It is surprising that so many students learn formal methods of solving equations without realizing what it is that they are finding. Many fail to grasp the simple idea that a root of an equation in x is a number which, substituted for x, satisfies the equation; or that a solution of a set of simultaneous equations in several unknown quantities consists of a *set* of values which, substituted respectively for the unknown quantities, will satisfy all the equations. In the ordinary processes of solution there is always danger of obtaining values of the unknown quantities

which are not roots; and hence the only safe plan is to test all results by substitution in the original equations. Thus, by the ordinary processes, one is likely to get

$$x = 1 \text{ or } -4/7$$

from the equation

$$\frac{6x}{x^2 - 1} - \frac{3}{x - 1} = 7,$$

but $x = 1$ is *not* a root, because it does not satisfy the equation.

This example leads to another matter. The idea should be strongly emphasized that *there is no such thing as division by zero,* that such expressions as $5/0$ and $0/0$ are meaningless, and that such expressions as $\frac{3}{x-1}$ and $\frac{x^2-1}{x-1}$ are meaningless when $x = 1$. There is no such number as ∞, and it is a pity that the symbol occurs in our books. To say that $5/0 = \infty$ has no meaning from any point of view whatever. And when we say that $\frac{3}{x-1} = \infty$ when $x = 1$, we are using figurative language which is unfortunate but which we have to make the best of while it remains in our text-books. What we mean is really quite simple. We are not considering the case of $x = 1$, but are letting x *approach* 1, and the fraction $\frac{3}{x-1}$ consequently becomes larger and larger. To put it accurately, however large a number be given us, we can take x so near 1 that the fraction $\frac{3}{x-1}$ will be larger than the given number. Similarly, we can say nothing whatever about the value of $\frac{x^2-1}{x-1}$ when $x = 1$, but we can say that as x approaches 1 the fraction approaches 2. When we say (figuratively) that the tangent of 90° is ∞, we mean first of all that this angle has no tangent. We also mean that we can take an angle so close to 90° that its tangent will be bigger than any number given us in advance.

In certain parts of the analytic geometry it would be greatly to the advantage of the student if he knew enough about quadratic inequalities to determine for what values of x a numerical

quadratic expression (such as $2x^2 - x - 3$) is positive and for what values it is negative.

In trigonometry a very troublesome deficiency on the part of many students is the lack of a *general* definition of the trigonometric functions. They do not know exactly what $\sin x$ means except for values of x between 0° and 90°. They should master a general definition that would hold for all angles, large or small, positive or negative.

The radian as a unit for measuring angles is not understood, as is shown by students asking why in some cases π stands for the number 3.1416 while in other cases it means 180°. Of course, π never means 180°, but means 3.1416 radians. The most important thing in this connection is the fact that when an angle is measured in radians, the length of a circular arc is equal to the angle subtended at the center multiplied by the radius. It is because of this fact that the radian replaces the degree entirely in the study of calculus.

A simple problem of frequent occurrence is that of finding the value of one function of an angle (except for the sign) when another function is given. Plenty of exercises, literal as well as numerical, should be given covering this point.

If possible, more time should be given to the solution of trigonometric equations. This is one of those subjects, theoretically simple but practically difficult, in which abundant practice is needed. The testing of roots in the original equation is especially important here, because the introduction of extraneous roots occurs so frequently.

And, finally, most of our students have only the vaguest notions about inverse trigonometric functions. These functions play an important part in the integral calculus. When one of two men working on a problem gets the result

$$\tan^{-1}\frac{5\tan\frac{x}{2}-3}{4}-\tan^{-1}\tfrac{1}{2}$$

and the other gets

$$\tan^{-1}\left\{2\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)\right\},$$

it does not even occur to them that their results may be the

same, but this is the case. To show that the one expression is equal to the other one is a more difficult problem of this kind than the students usually have to handle; but it is much easier than would appear at first sight, and they should know how to attack such a problem.

There are many students in engineering work who would have succeeded better in some other course; and the teacher of school mathematics has an important duty in this connection. He should strongly advise students who show no special aptitude for mathematics, and who have more ability along other lines, not to take engineering courses.

CORNELL UNIVERSITY,
ITHACA, N. Y.

In these near intimacies, we are ninety-nine times disappointed in our beggarly selves for once that we are disappointed in our friend; that it is we who seem most frequently undeserving of the love that unites us and that it is by our friends' conduct that we are continually rebuked and yet strengthened for a fresh endeavor.

We are all travellers in what John Bunyan calls the wilderness of this world—all, too, travellers with a donkey; and the best that we find in our travels is an honest friend. He is a fortunate voyager who finds many. We travel indeed to find them. They are the end and reward of life.—*Robert Louis Stevenson.*

SOME SUGGESTIONS ON DECREASING THE MORTALITY IN OUR GEOMETRY CLASSES.

By William R. Lasher.

First of all it is necessary for us teachers to get down to the level of our pupils or as near to that level as is possible. To do this we must recognize in the beginning that they have had very little that can be called preparation for the subject of logical geometry. They are largely rule-and-fact taught. They have learned one example and then worked the rest like it.

They come to us, however, with minds open to the great facts of history and of science. They are readily interested in the laws of nature and in the beauties of the outer world. They delight in real things. They are but little interested in metaphysics or philosophy. Their minds are too near the earth for these subjects. They have not yet acquired enough concepts to form a basis for philosophical thought.

We receive these young folks with appetites keen for the new and the wonderful and their first banquet at the geometric table is too often the enormous and awe-inspiring revelation "All straight angles are equal" elaborately carved into hypothesis and conclusion and served up with axioms and definitions galore. It is not strange that so many pupils fed in this manner experience an early nausea for the subject of geometry. To use their own phrase they "don't see any sense in it." The teacher is compelled to hold interest in the repast by continual reference to the choice viands that are to be served in the succeeding courses. This method is likely to be sadly lacking in efficacy. The pupil tires of the whole subject before he reaches that which is vital in it.

If the early facts in geometry or rather those facts which the logical arrangement of the work seems to require that we should place first in our texts were more interesting and striking; if we could, for instance, prove that straight angles are not equal, or that things equal to the same thing are not always equal to each other; if we could do something of this kind we should

*Read before the New York Section.

be far more likely to command and hold attention in the early stages of the geometry.

If we must give these basic facts upon which the logic of the geometry is founded so early, let us give them as facts and worry but little about the proofs of them. The ability to give logical demonstrations is the fruit of the study of geometry. We should not expect it, as we often do, in the very beginnings. If the pupil understands what straight angles are he will accept without question the fact that all such angles are equal. A knowledge of what a straight angle is is of importance to the beginner, but for him to be able to repeat what he thinks is a childish circumlocution of words, in order to arrive at the very evident conclusion that one straight angle is equal to another, is worse than useless.

Talk over the preliminary theorems with the pupil, show him their reasonableness, convince him of their truth and let them pass at that. If the fact that certain work in the first part of the book has not been rigidly demonstrated weighs heavily upon the conscience of the teacher, the pupil may be asked to return and demonstrate these theorems when he is able to see some reason why they should be demonstrated. Again we should not expect complete demonstrations even when we reach those theorems the demonstration of which is comparatively interesting. It is very easy in the early work in the geometry to sacrifice all chance to develop pleasurable interest to a desire to be thorough. Get a considerable number of geometric ideas before the class, give some new thought every day, but review and review again and again. Special and easy cases leading to the general case, numerical examples and illustrations, repeated drawings and simple constructions are of inestimable value in bringing the subject down within the pupil's range of vision.

Pupils must acquire a large number of ideas concerning the size, shape and areas of figures before proceeding to the work of logical demonstration. It is idle to expect children to philosophize about a subject when they have no knowledge of the fundamental ideas upon which the philosophy is founded. Early construction work in plenty is exceedingly helpful in this connection.

It must be very apparent to every teacher that the subject

matter laid down by the ordinary text-book is too great to be covered in a course one year in length. If the time is limited to one year, the work should be pared down so that it can be well handled. What is lost in breadth will be more than compensated for by increased efficiency and power in the matter covered. No real interest comes until the pupil begins to acquire a mastery of the subject. A great mass of only partially assimilated material is but dead weight to be sloughed off instead of becoming a part of one's mental bone and muscle. Simplify the course then, and simplify it until the pupils can conquer and not be conquered.

But even granted the best planned of courses and the most skillful of teachers we are still confronted with the problem of a considerable percentage of our geometry classes who cannot keep step with their fellows. It is also true that in no other high school subject is the "submerged tenth" so totally submerged, so absolutely helpless and such a dead weight upon the buoyancy of the teacher as in the subject of geometry. What to do with this large class of pupils who fall hopelessly behind early in the term and who continue on a clog to the wheels of progress and a source of unutterable worry and irritation to the conscientious teacher is a problem of absorbing magnitude.

The remainder of this paper will be given to a brief description of an effort to alleviate this difficulty that has been inaugurated by Mr. Feldman at Erasmus Hall.

About the middle of each term special classes are formed of the pupils who are failing in the work of the 2 A or beginning geometry. Such pupils are advised and urged to enter these classes but are not compelled to do so. In these classes the work is reduced to a minimum and the time for the course is lengthened to three terms instead of two. Consequently we have three grades denominated $2x$, $2y$, and $2z$ to distinguish them from the regular classes on programs and records. It is not necessary here to go further into the details of the method of organization. The arrangement of such classes involves some administrative difficulties but it is nevertheless perfectly feasible and we believe it is worth the trouble.

You will see of course that this plan involves the segregating of those pupils who are having difficulty with the geometry, an idea which in general is rather vigorously condemned.

If you will pardon personal allusion, and I think I may justly ask you to do so as this is an "experience meeting," I can best show you how the plan works by telling you something of my own experience with these classes.

In the middle of the last term but one, the head of our department informed me in a rather apologetic manner that mine was the only program into which one of the special classes could be fitted and asked me if I would take the class. This was the beginning of the experiment. Our chairman's manner showed that he felt that he was giving me a disagreeable task and I expect my manner showed that I fully concurred with this feeling.

I had previously had some experience with pupils who were failures in the subject of mathematics. I had taught classes made up entirely of "left backs," and I had also taught classes of pupils that had been gathered together with an almost absolute lack of the mathematical sense as a basis of classification. The memory of these classes and of the hard dragging work necessary to get any results at all from them was still exceedingly fresh in my memory. However, I went ahead with the class and before the end of the term I was able to say that there were few if any classes in geometry during the whole of my teaching experience that I had ever so thoroughly enjoyed. I do not for a moment wish to convey the impression that I felt that I had made skilled geometricians out of those who had formerly been adjudged failures and more or less hopeless ones, but I did feel that I had taught them something and I felt that they felt that they had learned. The spirit of the class was excellent, attention and responsiveness as good as or better than is found in the average geometry class. Now my experience is not unique. The other teachers of the department who have taught these classes for some time have the same feeling about them that I have. This is but the third term that we have used this plan. The pupils who started in the first of these special classes are now (June, 1911) completing the geometry. The plan must therefore be regarded as still in the experimental stage but we look upon it as a very gratifying experiment.

The success of the scheme so far, it seems to me, is very largely

due to the method of organization. The plan was put before the pupils not as a penalty for failure but as an opportunity to accomplish something they could not do in the regular way. That this method of presenting the matter was effectual is shown by the fact that six months ago when there arose so many difficulties with regard to programming that it seemed no special classes could be organized, the pupils besieged Mr. Feldman to have them organized, some even bringing their parents to emphasize the request. As long as we can keep up this spirit with regard to these classes I personally feel that they will be successful. If pupils are forced into them, assigned to them without a personal conference, if they become mere chain gangs for geometric convicts, they will lose a very large share if not all of their value.

At present they are an aid not only to those pupils who must go slowly in the geometry, but also to the better pupils, for they rid the sections of the dead wood which is always such an obstruuction to progress. I need hardly say to an audience of teachers that the removal of the failures from the classes takes a staggering burden from the shoulders of the teachers of the classes.

The presence of these classes in the school can often be used as a spur to induce pupils to keep up with the regular classes. We often say to a pupil: "Now we shall be glad to have you stay with the regular class if you can keep the pace. If you cannot, for your own sake you ought to go into the special class." We have had pupils go on under this sort of probation and pass the regular work of the term pupils who in the opinion of their teachers would have been flat failures without this incentive.

It must not be forgotten that in this scheme the work is adopted to the pupils. The subject matter is simplified and a longer time is allowed for its completion. The other special classes that I taught were expected to cover the same work in the same time as the regular classes. This is poor pedagogy. It is contrary to reason and to sense.

At present we have in these classes 166 pupils out of a total of 632 who are studying geometry. Of those in the second term or 2y classes 60 per cent. are passing in their work. Of

those in the third term or *2z* classes 45 per cent. are passing. The *2x* classes for the present term have just been organized, consequently no figures concerning them are available. The larger percentage of failures in the *2z* classes is accounted for by the fact that pupils are not "left back" in these classes. They are allowed to go steadily on till they cover the work whether they pass or not.

Considering the fact that these classes are made up of the slowest pupils in geometry we regard these figures as very satisfactory and as indicating that the scheme is a decided success so far at least.

ERASMUS HALL HIGH SCHOOL,
BROOKLYN, N. Y.

Of what shall a man be proud if he is not proud of his friends?—*Robert Louis Stevenson.*

Thou, O my love, ye, O my friends,—
The gist of life, the end of ends—
To laugh, to love, to live, to die,
Ye call me by the ear and eye!

SOME EXPERIMENTS TESTING THE MOST RAPID METHOD OF FACTORING THE TYPE $ax^2 + bx + c$.

By Fiske Allen.

For some time there has been considerable debate among the mathematics teachers of Horace Mann High School as to the best method of factoring the quadratic trinomial of the type $ax^2 + bx + c$. It was rather generally agreed that the so-called "cross-product" method is most easily made rational to the pupil and that the method of "splitting the middle term" is least easily taught; also that guessing is involved in all the methods. But it is insisted that either the "splitting-middle-term" method or "multiply-by-a" method is more scientific than the "cross-product" method because a definite system of guessing can be given for the first two which does not apply to the third. This last argument is denied, with no possibility of proof or conviction, but a disagreement upon the question of which method is most rapid was more easily tested and the following experiments made.

We had three sections of our first-year class taught by three different teachers and all were ready to begin studying this type of factoring on the same day. We assigned to class A the method of "splitting-middle-term," to class B the "cross-product" method and to class C the "multiplying by a" method. Instruction in the three classes was to begin on the same day and drill was to continue till the teachers agreed they were ready for a test. We agreed to select for the test the most difficult list on this type we could find in any text and the list of twenty-four problems found in Milne's "College Algebra" was selected. The pupils were told to solve as many as possible in exactly thirty minutes. The results of this test showed class A had solved an average of 8 problems, class B 15, class C 6.

Next, in order to test the relative abilities of the classes and also of the teachers, it was decided to hold two more tests on factoring types taught exactly alike to the three classes. For

this purpose we first selected the type $a^2 - b^2$, drilled on it the same length of time and on the same day gave the same test to the three classes. The results of this test showed class A 21.8, class B 15.3, class C 16. Our next ability test was on the types $a^3 \pm b^3$ and $a^5 \pm b^5$. This test showed class A 17.2, B 14, C 15.2.

These results were so conclusive that the opponents of the cross-product method set to work to find the weak points in the experiments. The most vulnerable point was the fact that the teacher of class A was not enthusiastic for the method he was teaching but preferred the cross-product. And the point was also made that the problems chosen were too easy and generally "first-guess" problems. It was asserted that the cross-product method would fail of the rapidity of the other methods if larger numbers with more possible factors were given. To obviate the first of these objections we induced a former teacher in Horace Mann and an enthusiastic advocate of method A to teach class A for four days. He was to have one day to become acquainted with the class, the other teachers giving but three days to drill. At the close of that period another test was given. . The problems were made to order, since none of sufficient difficulty could be found in any text and all first guess problems were eliminated. The list was submitted to each teacher and any problem peculiarly adapted to another teacher's method was eliminated. Because of the difficulty of the problems the results of this test were very low, as follows: Class A 4.7, B 6, C 3.5.

This test was given the last day preceding the Easter vacation. Immediately following it two more tests were given on the same list of problems with results as follows: First test—A 3, B 5.6, C 2.8; second test—A 3.7, B 5, C 2.9. It was then decided to give two more ability tests, the first on the type $x^4 + x^2y^2 + y^4$. In this test class A solved 15, B 13, C 18.4. Of the type factored by the "Factor Theorem" A solved 3.5, B 4.8, C 2.1.

Averaging the results of the various tests we find that on the type $ax^2 + bx + c$ class A solved 4.9, B 7.9, C 3.8. On the ability tests the average was class A 14.4, B 11.8, C 12.9. Considering the relative ability of the three classes as shown by the ability tests the results of the $ax^2 + bx + c$ type *should have*

been: A 9.6, B 7.9, C 8.5. Comparing the *actual* results with those which should have been shown had all classes been taught the cross-product method we find the method of "splitting the middle term" barely half as efficient as the other, and "multiplying by *a*" little more than one third as efficient.

We do not regard this single experiment as at all conclusive, but it points so emphatically to one conclusion that we shall be much interested in having our results corrected or corroborated by similar experiments others may make.

HORACE MANN HIGH SCHOOL,
NEW YORK CITY.

Purge out of every heart the lurking grudge. Give us grace and strength to forbear and to persevere. Offenders, give us the grace to accept and to forgive offenders. Forgetful ourselves, help us to bear cheerfully the forgetfulness of others. Give us courage and gaiety and the quiet mind. Spare to us our friends, soften to us our enemies. Bless us, if it may be, in all our innocent endeavours. If it may not, give us the strength to encounter that which is to come, that we be brave in peril, constant in tribulation, temperate in wrath, and in all changes of fortune, and down to the gates of death, loyal and loving one to another.—*Robert Louis Stevenson.*

NOTES ON THE TEACHING OF MATHEMATICS IN AN OBERREALSCHULE.

By M. O. Tripp.

The *Realschulen* of Germany originated in a reactionary movement against the humanistic studies. One fundamental idea in the starting of the *Realschulen* was preparation for commercial professions, while the humanistic institutions prepared for the learned ones. In the course of time, however, the *Realschulen* have enlarged their curriculum to meet the demands of general culture.

The *Oberrealschule* with its wide range of subjects gives a course of nine years, the same length as that of the humanistic *Gymnasium*. Between these two classes of schools there is the *Realgymnasium*, with Latin but no Greek, and having a nine years' course also. Girls are prepared for the university in *Lyceen* which give a three years' course following graduation from the *Höhere Mädchenschulen*.

Pupils are generally at least nine years of age when they enter the lowest class in the *Oberrealschule*. They must possess the following qualifications: ability to read German or Latin print, a legible handwriting in either German or Latin script, readiness in taking down simple dictation, ability to carry out the four fundamental operations of arithmetic, and some knowledge of biblical history. The subjects taught in the *Oberrealschule* are religion, German, French, English, history, geography, mathematics, natural science, drawing, penmanship, singing and gymnastics. The average age of graduation is about twenty years. Graduates from the *Oberrealschulen* and *Gymnasien* are granted admission to German universities on practically the same basis. In consequence of government control and centralization of authority there is almost absolute uniformity in the courses of study, so that when an observer has visited one school of a particular kind the work may be considered as typical of all schools of that class.

The notes which form the basis of this article were taken at

the *Kaiser Wilhelm II Oberrealschule* in Göttingen where I carefully observed the methods of instruction in various classes, after having obtained special permission from the Prussian Minister of Ecclesiastical, Instructional and Medicinal Affairs. The work in mathematics is placed under the heading *Rechnen und Mathematik* and covers a wide variety of subjects from elementary arithmetic through analytic geometry to a little calculus and descriptive geometry. In mathematics and physical science the *Oberrealschule* gives a number of hours greater than any other secondary school. More time is devoted to mathematics than to any other subject in the curriculum, French being a close second. Religion and drawing each take up about one-third as much time. Mathematics which is taught four hours per week for the first two years, six hours per week for the next three years, and five hours a week for the last four years occupies a trifle more than one-sixth of the entire time. The total time of instruction in all subjects varies from thirty to thirty-eight hours (*i. e.*, fifty-minute periods with ten minutes intermission) per week. Such a thing as a study period is conspicuously absent. It is quite common to have two subjects taught simultaneously and even during the same period. In one class half the period was given to descriptive geometry and the other half to calculus. The length of vacation is about the same as in the secondary schools of this country, but recitations are held six days in the week, so that more time is really given to school work.

Perhaps the most striking feature of the mathematical work is the limited use of blackboards. In the recitation rooms there was usually a small blackboard where only one person could work at a time, and even then much work was carried on orally which in American schools would have been illustrated at the board. In a long discussion of elementary processes in algebra, students of the *Untertertia* (corresponding approximately to our eighth grade) showed unusual facility in handling orally algebraic expressions, such as those ininvolved in factoring $a^6 - b^6$ and $a^5 - b^5$. The discussion of powers of 10 ran up as high as quadrillions. The only blackboard work during the recitation was an application of the simple equation to the evaluation of a circulating decimal. In

another class logarithms were discussed without using the blackboard. In geometry and trigonometry a figure was usually drawn on the board and then the whole class took part in the discussion of the theorem or problem. I did not see any demonstrations written on the blackboard. The teacher questions continually, giving help whenever necessary to secure prompt and complete statements in reply. A large amount of time is spent in reviewing previous theorems. In talking with the instructors I found that they did not want any more blackboard space in their recitation rooms, since they believe that the mental training developed with one small blackboard and much oral discussion is greater than when there is a large amount of written work.

Text-books play a rather unimportant part in the mathematical instruction, although students are required to have them. The instructors do not follow a text closely and, in general, pay no attention to it except for exercises. One instructor in analytic geometry, however, told me that occasionally he had his students study the text carefully. I found the class in trigonometry using a text by Holzmüller and tables by Schlömilch. In one class I saw a copy of Bardey's *Aufgabensammlung*. The Kambly texts are more widely used in Germany than those of any other author. They differ from our American text-books, since the elaboration of detail is left to the individual teachers, and the subject is given in outline; *e. g.*, the book on *Planimetrie* covers, in this way, what we give in plane geometry, plane trigonometry and also some solid geometry. The aim of the teacher is not to follow the text, but rather to make the text follow his work by way of supplementing, thus furnishing exercises which otherwise would necessitate loss of time in dictation. Notwithstanding the little stress placed on text-book work the instructors do not fall into the habit of lecturing. Instructors are not free to introduce a new text. This can be done only by higher authority.

Home work does not play so important a part as with us. It is generally arranged so that the preparation of lessons at home shall not occupy more than half as much time as the recitation period. In each class only one exercise was given out for home work. The large amount of time devoted to

recitations enables the instructors to spend much time in review and in the solution of exercises. The recitation period is also a period for the acquisition of new knowledge and hence this mode of instruction has been aptly termed the study-recitation method.

The method of conducting a recitation, as it prevails at Göttingen, is illustrated by the following lesson in analytic geometry on the parabola.

I. *Discussion of the equation of the parabola.* This equation was written in the form $y^2 = 2px$. The instructor then drew a figure on the board and asked questions about the parameter $2p$, directrix, etc. A student derived the equation of the parabola, explaining at the same time that he worked at the board. During this time the instructor repeatedly asked questions, which were answered as well by students in the seats as by the one at the board. Attention was called to the fact that as x increases y does; also, and by questioning it was developed that to one value of x there are two of y, and to one of y only one of x. At times students were slow in answering.

II. *Condition that a point is (1) inside, (2) outside, (3) on the parabola.*

(1) The instructor placed a point P inside the parabola and proceeded to quiz concerning it, while the students worked at their seats. Meanwhile the instructor placed results on the black-board. In this way there was developed for the point (x, y) within the parabola

$$(x + p/2)^2 - (x - p/2)^2 > y^2$$

or

$$2px > y^2.$$

The instructor stated the result: If a point (x, y) lies inside the parabola then is $2px > y^2$.

(2) The point outside the parabola was considered and the relation

$$(x + p/2)^2 < (x - p/2)^2 + y^2$$

or

$$2px < y^2$$

was developed in the same way as before. This time, however,

a student was asked to state the result: If a point (x, y) lies outside a parabola then is $2px < y^2$.

(3) The instructor said "when $y^2 = px$ we are absolutely sure that the point (x, y) lies on the parabola." Attention was called to the fact that the parabola is the geometric locus of $y^2 = 2px$. The three results were then placed together, as a test whether P is in, on or outside the parabola.

The correspondence between the algebraic and geometric considerations of the parabola were discussed. The construction of points of the parabola with ruler and compasses followed. The equation $y^2 = 2px$ was read as a proportion by students and the use of the proportion was explained. One student confessed that he did not understand and was given the required assistance by the instructor.

III. *Computation of areas of parabolic sections cut off by perpendiculars to the axis.* From previous proof it was stated that the area is equal to two thirds of the triangle formed by the perpendicular and the tangents at its extremities. Attention was called to the fact that the subtangent is bisected at the vertex, and then the area was expressed in the form $\frac{2}{3}x\sqrt{2px}$. The latter expression was then stated in words. The instructor told the students not to memorize this result. He spoke of the irrationality appearing in the expression and emphasized the fact that it is a different kind of irrationality from that of the Ludolphian number.

IV. *Consideration of the area cut off by the latus rectum.* A student was required to give a geometric interpretation of the expression $\frac{2}{3} \cdot p/2\sqrt{2p \cdot p/2}$ in terms of the latus rectum.

V. *Solution of exercises.* Students were required to solve the following exercise: In a parabola the parameter is 10 cm. How great is the section corresponding to the abscissa $x = 15$? The instructor gave the equation of the parabola and asked about the magnitude of the parameter. Finally an exercise taken from the text, Knop's "Analytic Geometry," was considered. The students did not answer well when asked how to attack the problem, showing that they had not fully comprehended the theory.

It is, of course, true that the plan of teaching with a small blackboard and a text merely to supplement the recitation work

produces and requires stronger teachers than an abundance of blackboard work and slavish obedience to the text-book. It is doubtless true that, in this country, teachers often spend too much time in mere routine blackboard work and not enough in careful questioning and discussion. It is highly probable that our better teachers, with an adequate blackboard equipment and a well selected text, do fully as good work as the Germans. There is, in fact, a decided advantage in training students to the use of books so that, to a certain extent, they may become independent.

The success of the Germans is, I believe, due neither to the lack of blackboard work nor to the little stress laid upon text-books; but rather to the highly trained teachers who give instruction in the secondary schools. Ph.D. men are quite common in the *Gymnasien* and *Oberrealschulen*. In the Göttingen *Oberrealschule* eight of the twenty teachers were doctors. I am informed that in some institutions the doctors are in the majority. The status of the teacher with his permanent position and pension is considerably above that of the American teacher. Competition for educational positions is keener than with us because the German teacher, as compared with other members of society, is much better paid than in America.

COLLEGE OF THE CITY OF NEW YORK,
NEW YORK CITY.

NEW BOOKS.

Advanced Algebra and Trigonometry. By W. C. BRENKE. New York: The Century Company. Pp. 350.

Professor Brenke has combined and correlated in one book what is usually covered by college freshmen in algebra and trigonometry. Correlation in secondary mathematics has been tried and the experiment with the college work will be watched with interest. The value of the book for review classes in these two subjects could hardly be questioned. Considerable space is devoted to graphic methods and the idea of the derivative is introduced and applied to some of the simpler functions.

Plane and Solid Geometry. Revised by GEORGE WENTWORTH and D. E. SMITH. Boston: Ginn and Company. Pp. 480. $1.30.

This revised edition is certainly an improvement on the old one. Considerable new matter has been added and the whole rearranged.

An Introduction to Mathematics. By A. N. WHITEHEAD. New York: Henry Holt and Company. Pp. 256. 75 cents.

Mr. Whitehead sets out not to teach mathematics, but to "enable students from the very beginning of their course to know what the science is about and why it is necessarily the foundation of exact thought as applied to natural phenomena." It is just because mathematical ideas are abstract that they supply what is wanted for a scientific description of the course of events, freed from reference to particular persons or particular types of sensation. From this starting-point the author proceeds to explain the true inwardness of variables, dynamics, symbols, generalizations of numbers, coördinate geometry, conic sections, functions, periodicity, trigonometry, the differential calculus, and geometry. It is an admirably clear exposition, illustrated throughout with diagrams; and one which every teacher of mathematics will want to read. Every student should be familiar with its contents.

Vocational Mathematics. By GEORGE WENTWORTH and D. E. SMITH. Boston: Ginn and Company. Pp. 88.

The Hindu-Arabic Numerals. By DAVID EUGENE SMITH and LOUIS C. KARPINSKI. Boston: Ginn and Company. Pp. 160. $1.25.

Although it has long been known that the numerals ordinarily employed in business, and commonly attributed to the Arabs, are not of Arabic origin, and although numerous monographs have been written concerning their derivation, no single work has yet appeared in which the complete story of their rise and development has been told. In the

preparation of this treatise the authors have examined every important book and monograph that has appeared upon the subject, consulting the principal libraries of Europe as well as America, examining many manuscripts, and sifting the evidence with greatest care. The result is a scholarly discussion of the entire question of the origin of the numerals, the introduction of the zero, the influence of the Arabs, and the spread of the system about the shores of the Mediterranean and into the various countries of Europe.

The work is illustrated with numerous facsimiles from early inscriptions and manuscripts, most of which have not heretofore been published in connection with this subject, and all of which contribute to a very marked degree to an understanding of the problem.

The Courtis Standard Tests in Arithmetic. By S. A. COURTIS. Detroit, Mich., 441 John R. Street. Pp. 40. Manual of Instructions, .25 cts.

Education as a science must always be seriously handicapped by the difficulty of measuring results. Only of recent years has the desirability of measuring educational results been advocated. Even now, so many factors enter into the problem, factors largely of a psychological character, that most of us would hesitate to have the results of our teaching tested, save in a very general way. We are beyond the old notion that examinations are sure tests of effective teaching, and the progressive teacher of today would never admit that his great ambition was to have his pupils successfully pass State Board or College Entrance Tests. Nevertheless, the examination idea, which is not fundamentally wrong, must be postulated in all efficiency tests in mathematics, and efficiency tests are so reasonable in most walks in life, that it seems most probable that there is such a thing as measurable efficiency in mathematical teaching.

Professor S. A. Courtis, head of the department of mathematics and science in the Detroit Home and Day School, has for several years made use of comparative tests for purposes of supervision. During this time he has developed a system of "Standard Tests" which are suitable to test ability in arithmetic as taught in the years preceding the high school. The author suggests the following ways in which they may be useful: as comparative tests, given to many classes under identical conditions; to measure growth during any school year; to show the teacher the weak and strong places in his teaching; to enable principals and superintendents to compare teachers, grades, schools, and even school systems.

Fourteen separate tests, or papers, have been devised—all confined to the four fundamental operations, including easy applications. Each test is timed so that the most rapid pupil will not have completed the work within the limit. Mr. Courtis recognizes that ability in arithmetic is exceedingly complex. Not claiming that his analysis is complete, the author aims to measure the following component abilities: control of knowledge of the fundamental combinations in each of the four operations; control of the symbols, processes, and forms of each of the four

operations; rate of motor activity; ability to "borrow and carry"; attention to mechanical details; recognition of the particular operation required (reasoning); and the ability to use all the above in the complex situation of abstract examples, or two-step problems.

Explicit directions are given for giving the tests and for scoring the results. Uniform blanks are furnished on which to tabulate the results and teachers are encouraged to send these records to the author's headquarters where Professor Courtis is making a careful study of the material thus collated. The tests are printed in convenient form for the pupil's use, and are sold without profit to those who will co-operate with the author in these investigations; to independent workers the cost is also very moderate.

The thoroughness with which Mr. Courtis has standardized his tests is an indication of the careful study he has given to the problem. He has met the criticisms that generally apply to such pedagogic studies, and his directions to teachers, if carefully followed, should remove from the individual experiment much of the error resulting from peculiar personal conditions and environment.

Some teachers will object to these tests on the ground that they are too easy even for young children. In this connection, the following observation of the author is interesting: "Experience has shown that the mistakes, except for the zero combinations, are very few. Ignorance is shown, not by mistakes, but by reduced speed. The answers written are usually right except in the very lowest grades." Throughout the tests great emphasis is placed on speed. It is a grave question whether speed and accuracy, and speed and mathematical power go hand in hand. We have seen speedy computers both inaccurate and poor thinkers; conversely, slow computers are sometimes accurate, and strong reasoners. Speed is a very valuable accomplishment these days, and accuracy is essential; but most valuable of all is reasoning power. The inculcation of this important habit is the chief business of the teacher of mathematics, even of the teacher of arithmetic. Will not Professor Courtis put more stress on *reasoning* when devising additional tests to supplement this useful collection?

Elements of the Differential and Integral Calculus (Revised Edition). By WILLIAM ANTHONY GRANVILLE. Boston: Ginn & Company. Pp. 463, $2.50.

While retaining the fundamental characteristics of the original edition, this new edition will be found thoroughly modern and more teachable than before. Besides the changes which may be attributed chiefly to the recent developments in the study of the calculus, special notice may be taken in the Revised Calculus of the abridgment of introductory material; the introduction of biographical sketches of the leading men connected with the history of the calculus; and the addition of a large number of examples without answers, miscellaneous examples, and simple, practical problems, based on knowledge that all students of the calculus should have at their command.

The Principles of Scientific Management. By FREDERICK WINSLOW TAYLOR. New York: Harper & Brothers. Pp. 144.

In the mechanical world much seems to have been done in the way of lessening the cost of production by increasing the efficiency of the laborer. This efficiency has been brought about by careful experimental studies as to the best methods and conditions for the work. The author of this book has taken a very prominent part in the improvements and presents in this volume many instances and figures showing what has been accomplished. Teachers should give it a careful reading and they will then see that similar investigations into the nature and methods of their work might lead to much greater efficiency.

Practical Algebra. Second Course. By Jos. V. COLLINS. New York: American Book Company. Pp. 313.

This book consists of two parts, a *review* of the First Year Course, followed by *additional chapters* on more advanced topics, the whole intended to provide the preparation for entrance to colleges of liberal arts. The author aims to unify arithmetic, algebra, geometry and applied mathematics and places much stress on the use and interpretation of *formulas*.

NOTES AND NEWS.

THE Annual Meeting of the Association will be held on December 2, at Columbia University, New York City.

SECONDARY SCHOOL MATHEMATICS

By ROBERT L. SHORT and WILLIAM H. ELSON,

Head of Dept. of Mathematics, Cleveland Technical High School

Superintendent of Schools, Cleveland, Ohio

A course in mathematics for the first two years of high school work. Pedagogically, the book is unique and correct. The various subjects of mathematics are so interwoven that the pupil does not tire of his work. The variety adds interest and strength. Attention is called to the following :

1. An early study of numbers both by factors and decimal composition prepares for algebraic number and an understanding of polynomials.

2. Constant use of the equation, in both algebra and geometry, gives the pupil a reason for and an application of his algebra.

3. Teaching of the subjects simultaneously avoids much needless repetition, and keeps each subject in constant use.

4. The introduction of sine, cosine, and tangent ratios is of value in the study of theorems relating to equality, equivalence, and areas, and is a great saving in much of the computation work.

5. Lists of problems of applied mathematics relating to the shop and to household arts are helpful to those teachers wishing to emphasize the technical side of mathematics, and may be assigned or omitted at the discretion of the teacher.

6. Lists of classified theorems facilitate the work of both teacher and pupil.

7. Problems, real, not difficult, full of interest, hold the pupil's attention and urge him on in his work.

BOOK I. (First year work) Arithmetic, algebra through fractional equations and proportion, straight line geometry to proportion.
Half leather. viii+182 pages. Price, $1.00.

Book II. (Second year work) Arithmetic, algebra through quadratic equations and logarithms, geometry of the circle, proportion and areas, trigonometry of the right triangle. Half leather. vii+180 pages. Price, $1.00.

D. C. HEATH & CO.

BOSTON NEW YORK CHICAGO LONDON

20 Reasons Why You Should Purchase

The No. 12 Model

HAMMOND

1. Visible Writing.
2. Interchangeable Type.
3. Lightest Touch.
4. Least Key Depression.
5. Perfect and Permanent Alignment.
6. Writing in Colors.
7. Least Noise.
8. Manifolding Capacity.
9. Uniform Impression.
10. Best Mimeograph Work.
11. Any Width of Paper Used.
12. Greatest Writing Line.
13. Simplicity of Construction.
14. Greatest Durability.
15. Mechanical Perfection.
16. Back Space Attachment.
17. Portability.
18. Least Cost for Repairs.
19. Perfect Escapement.
20. Beauty of Finish.

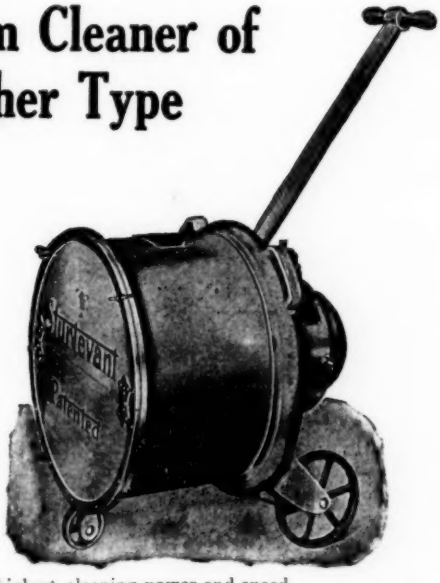
WRITE FOR CATALOG

The Hammond Typewriter Company

69th to 70th Streets, East River

NEW YORK, N. Y.

A Vacuum Cleaner of a Higher Type



A machine of the highest cleaning power and speed.

Yet it will not damage the most delicate fabric. It entirely does away with the useless and injurious *excess suction* that has been the serious objection to vacuum cleaning. This is the greatest advance that has been made in *household* vacuum cleaning.

It is an entirely new conception of powerful simplicity. It has none of the valves, gears, bellows, diaphragms, which are responsible for the endless repairs and short-lived efficiency of so many vacuum cleaners.

It is consequently the easiest vacuum cleaner to use and to take care of. It may be counted for long unimpaired service.

No other vacuum cleaner has so wide a range of usefulness. No other vacuum cleaner so completely lifts the daily burden of house-cleaning.

There are lower-priced vacuum cleaners but none that can possibly give so large a return for the money invested.

THE Sturtevant

ELECTRIC VACUUM CLEANER

Both suction and blowing—Largest capacity of any portable cleaner. Will clean a large house without emptying—Rubber tires—Easily carried up and down stairs—Ten special cleaning tools—Price, delivered anywhere in the United States, **$130.**

SEND FOR BOOKLET MT

The Sturtevant machine may be seen at any of our branches

B. F. STURTEVANT CO., Hyde Park, Mass.

50 Church Street, New York; 135 North Third Street, Philadelphia; 530 South Clinton Street, Chicago; 329 West Third Street, Cincinnati; 711 Park Building, Pittsburg, Pa.; 1006 Loan and Trust Building, Washington, D. C.; 34 Oliver St., Boston; Metropolitan Building, Minneapolis; 423 Schofield Building, Cleveland; 1108 Granite Building, Rochester, N. Y.; Fullerton Building, St. Louis; 456 Norwood Avenue, Buffalo, N. Y.; 36 Pearl Street, Hartford, Conn.

Little Differences

indicate the boundary between the ordinary and the superior. The designs of our jewelry are a little more exclusive than any others, the workmanship a little finer, the quality of the gold and the gems a little better—little differences you will certainly appreciate.

ROLLED GOLD PINS, LOCKETS and SLEEVE LINKS

in quaint new styles for the economical purse.

STETSON & CROUSE

127 South Salina Street SYRACUSE, N. Y.

THE NEW ERA PRINTING COMPANY

LANCASTER, PA.

is prepared to execute in first-class and satisfactory manner all kinds of printing and electrotyping. Particular attention given to the work of Schools, Colleges, Universities, and Public Institutions.

Books, Periodicals

Technical and Scientific Publications

Monographs, Theses, Catalogues

Announcements, Reports, etc.

All Kinds of Commercial Work

(*Printers of the Bulletin and Transactions of the American Mathematical Society, etc., etc.*)

Publishers will find our product ranking with the best in workmanship and material, at satisfactory prices. Our imprint may be found on a number of high-class Technical and Scientific Books and Periodicals. Correspondence solicited. Estimates furnished.

THE NEW ERA PRINTING COMPANY

"SWAN SAFETY"

The Fount Pen of Quality

Never Blots—Never Leaks—
Always Writes

THE SWAN SAFETY

Embodies the three great essentials, never leaks, never blots, always ready for use. A "Swan Safety" can be carried in any position—pocket or bag—without fear of leakage.

Prices $2.50 up.

Ask your Stationer or Jeweler to show you a "SWAN SAFETY," or write us for illustrated catalogue.

BECAUSE

The "Screwdown Cap" positively prevents leaking

The "Ladder Underfeed" and Gold Top Feed control the supply of ink, supplying the exact quantity—no more—no less.

The 14 Kt Gold Pen nibs are pointed in the highest grade pen made.

MABIE TODD & CO.

17 Maiden Lane, New York 209 S. State St., Chicago

LONDON TORONTO PARIS BRUSSELS